Exercises #1

•**Problem 1.** Compute the first 768 digits of π. Anything remarkable? Try a few digits more for good measure.

•**Problem 2.** Compute $(2143/22)^{\frac{1}{4}}$.

•**Problem 3.** The masses of the electron and protron are $m_e = 0.51109991(15)$ MeV and $m_p = 938.2723(3)$ MeV. Find an approximation of the form

$$\frac{m_p}{m_e} \approx 2^n 3^m \pi^p$$

with positive integers n, m and p. There is a solution good to 4 digits.

•**Problem 4.** The electron's magnetic moment is $\mu = 1.00115965219(1)e\hbar/2m_e$. A certain crank who posts to the Usenet "predicts" that the dimensionless constant which appears there should be

$$1 + \frac{1}{2}(\frac{\alpha}{\pi}) - \frac{1}{3}(\frac{\alpha}{\pi})^2 + \frac{1}{4}(\frac{\alpha}{\pi})^3 - \frac{1}{5}(\frac{\alpha}{\pi})^4 + \frac{1}{6}(\frac{\alpha}{\pi})^5,$$

where the fine structure constant is $\alpha = 1/137.0359895(61)$. Could he be right?

•**Problem 5.** One way of getting π is via the identity $\pi = 4\tan^{-1}(1)$, where the arctan can be developped in a series

$$\tan^{-1}(x) = (1 - \frac{1}{3} + \frac{1}{5} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}.$$

How many terms do you need to get 3 digits of π?

•**Problem 6.** Another such identity is

$$\pi = 16\tan^{-1}(\frac{1}{5}) - 4\tan^{-1}(\frac{1}{239}).$$

How many terms in the Taylor series do you need to get 20 digits of π?

•**Problem 7.** One last numerical accident. How close is $\exp(\pi\sqrt{163})$ to an integer? You may find the following incantation of help:

```
In[1]:=          NumberForm[%,ExponentStep->33]
```

•**Problem 8.** Find the roots of the equation

$$x^5 + 8x^4 - 72x^3 - 382x^2 + 727x + 2310 = 0.$$

Do this (a) with `Solve[]` and (b) with `Factor[]`.

•Problem 9. Solve the equations

$$\exp(x) = x$$

and

$$\exp(x) = -x$$

.

•Problem 10. Here are a set of linear equations which might arise, e.g. in a multi-loop circuit problem.

$$\begin{aligned} 2I_1 + 3I_2 - 4I_3 &= 5 \\ 3I_1 - 2I_2 + 4I_3 &= 6 \\ 4I_1 + 1I_2 - 4I_3 &= 7 \end{aligned}$$

Solve for the currents I_1, I_2 and I_3 using `Solve[]`.